Circulant Preconditioning of the Fast Gradient Method for Predictive Control

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Recently, Model Predictive Control (MPC) has grown in popularity due to its ability to incorporate operating constraints in the computation of an optimal control action. At its core, MPC solves an optimization problem to determine the next control action, which for the Constrained Linear Quadratic Regulator (CLQR) is a Quadratic Program (QP). This popularity has led to MPC being implemented on smaller processors and FPGAs that do not contain hardware to accelerate floating-point computations and that can take advantage of many parallelization opportunities in the problem structure and algorithms.

We present a preconditioner for the Fast Gradient Method (FGM) when the FGM is used to solve the condensed linear MPC problem. We exploit the block Toeplitz structure in the Hessian of the linear MPC problem's QP to create a block circulant preconditioning matrix that is easy to compute and that can accelerate the convergence of the FGM by up to 9x. To preserve the parallelization opportunities inside the FGM's projection step, we symmetrically precondition the Hessian and restrict the circulant preconditioner to be block diagonal, meaning the overall preconditioner can be defined by a single matrix $L \in \mathbb{R}^{m \times m}$, (where m is the number of inputs to the system being controlled).

We define A and B to be the system's state transition and input matrices, respectively, K to be a prestabilizing controller for the system, and P to be the solution to the discrete algebraic Ricatti equation or the discrete Lyapunov equation for the system. The blocks of the preconditioner can be computed in closed form, with L given by the lower-triangular Cholesky decomposition of M with

$$M := B'PB - B'K'R - RKB + R.$$

We show in numerical examples that this block circulant preconditioner has equivalent performance to the start-of-the-art preconditioner computed using a Semidefinite Program (SDP), giving between a 2x and 9x speedup for the FGM. The computation of the proposed preconditioner is orders of magnitude faster than the SDP-based preconditioner, with our preconditioner computed in milliseconds for large MPC problems instead of the minutes required for the SDP-based preconditioner.